

Finally, the one-dimensional-strain analogue of the Murnaghan equation

$$\sigma = \frac{A}{B} [(V_0/V)^B - 1]$$

can be expanded to give

$$\sigma = A \gamma [1 + 1/2 (B + 1)\gamma + 1/6 (B + 1)(B + 2)\gamma^2 + \dots] \quad (2.23)$$

Equating the derivatives up to second-order, we have

$$c_{11} = \rho_0 a^2 = A \quad (1st \text{ order})$$

$$-(3 + \frac{c_{111}}{c_{11}}) = 4b = (B + 1) \quad (2nd \text{ order})$$

Evaluating the parameters A, b, and B from these equations gives

$$\text{X-cut: } A = \rho_0 a^2 = c_{11} = 8.68 \times 10^{11}$$

$$b = -0.15$$

$$B = 1.6$$

$$\text{Z-cut: } A = \rho_0 a^2 = c_{33} = 10.575 \times 10^{11}$$

$$b = 1.177$$

$$B = 3.71$$

With these values all three expressions have the same slope and curvature at zero stress. The predicted stresses for various compressions are shown in Table IV.

That the closed form expressions are approximate is hardly surprising inasmuch as they are both empirical with no known physical basis. Their value is that they both are two-parameter functions that have physically reasonable shapes, and they are therefore convenient for interpolation and extrapolation when experimental information is lacking.

Knopoff's suggestion that, because of the arbitrariness in the definition of strain, alternative definitions may prove more suitable for representing constitutive relations would seem to be worthy of further consideration. However, some guidance from physical reasoning is necessary to provide any degree of generality to a given definition.

TABLE IV
Stress-Kbar

V/V ₀	X			Z		
	Murnaghan	Linear U _s - U _p	Finite Strain	Murnaghan	Linear U _s - U _p	Finite Strain
0.99	8.6	8.8	8.7	10.7	10.8	10.8
0.98	17.2	17.5	17.5	21.8	22.2	22.3
0.97	25.8	26.3	26.5	32.0	34.0	34.5
0.96	34.4	35.0	35.9	47.1	46.5	47.6
0.95	42.7	43.7	45.7	60.3	59.7	61.6
0.94	51.2	52.5	56.1	73.9	73.5	76.5
0.93	59.0	61.3	67.1	87.9	88.0	92.6
0.92	67.9	70.0	78.9	103.4	103.0	109.7
0.91	76.2	78.8	91.4	119.6	119.0	127.9
0.90	84.4	87.6	104.7	136.4	135.8	147.3